CHAPTER 7 Summing up

That numbers have distinct psychological properties has been demonstrated by several experimental measures. Some numbers are meaningful, while others are less so. Some are liked, while others are not. Certain numbers are easy to remember, but not to associate to. Four general factors were found to underlie the obtained distinctions. Briefly, these are psychological frequency, general positive appreciation, exclusiveness of associations and specific affective meaning.

During the preparation of this project we reasoned that certain objective attributes of numbers should influence their psychological representation. Tabled numbers, for instance, have more connections within the studied set of hundred than have non-tabled numbers. Such an advantage should express itself in the nature of these numbers' mental associations, as well as in their being better known in other respects. That magnitude, also, must be important was evident from the literature. We did not know, however, how such different attributes of numbers would influence people's performance in the tasks we meant to test them with. Neither did we know how the workings of such attributes could be understood in terms of mental structure.

Some of the differences obtained in the experiments reported in this book can be explained just by looking at the mathematical properties of the numbers concerned. This is well illustrated by the m -variable, which measured how many associations people could produce, in thirty seconds, to a given stimulus number. That 12 and 50 scored much better than 67 and 13 may just reflect their different mathematical properties. Since 67 and 13 have no natural divisors, and producing divisors was a frequently used option, these numbers are at a relative disadvantage. From other experiments it is clear that 13 is not "badly known" as such. That people found it difficult to relate 13 to other numbers can - given the apparently limited number of options available to our subjects - be predicted from its being prime.

A quite different type of non-mental attribute is "number of digits". People sometimes wrote down one or both of a number's two digits as associations. When this particular response strategy is used, a number's m -score will also be a function of its being one- or two-digit, making m a mixed measure of deep and superficial elements of knowledge. The tendency to use such superficial response options may also be responsible for the relatively long RTs and high proportion of non-responses (omissions) to the single-digit stimulus numbers in Experiment 1 of Chapter 3. Though subjects' preferred associations were of a more semantic type - a preference which was strengthened by the easier instruction used in Experiment 2 - it is evident that different types of objective attributes influenced the outcomes of the association experiments.

It is equally evident that people's general preference for a certain response type, in this case divisors, can be counteracted by their lack of readily available knowledge. Not all tabled numbers appear to be well known as such. This was demonstrated by several measures, such as associative content, associative agreement between subjects and speed of responding. In general, larger numbers seem to be less well known in terms of the preferred multiplicative relationships, showing that numerical magnitude puts constraints on rule driven associative performance.

No psychologist is needed to explain why people name 3 as an association to 9. The obvious reason is that these two numbers are in fact related, making the mental connection a rational one. Why these same subjects fail to handle other, similarly related, numbers in an equally rational manner is a more interesting question. It could be answered, though, by pointing out that small-sized and large-sized multiplication problems are not equally well taught. The discovery of Ashcraft and his colleagues that small size problems occur more frequently in primary school arithmetic texts seems to offer a good explanation of differences in meaningfulness and commonality within the group of tabled numbers.

The finding that certain objective properties also influence people's feelings about numbers suggests, however, that a concept's psychological meaning is not sufficiently described by listing its well-known numerical connections. In Chapter 4 it was demonstrated that numbers with many associations were judged to be relatively "good", while numbers with few associations were judged to be relatively "bad". Also, even numbers were less frequently cited as "unpleasant" or "excitable" than were odd numbers. Though such judgements may ultimately reflect certain objective properties of numbers, they cannot directly be explained from them. In those experiments people were not asked to be rational about numbers, but to observe their own minds' subjective signals. A number's being good or excitable is not taught in school. It does not necessarily follow from any of its objective attributes. Nonetheless, these concepts were found to describe some shared feelings about at least some numbers. Interestingly, these feelings could be traced to the same dimensions that have been found to underlie feelings about words.

It is on the experimental measures of frequency that the natural system's limitations and preferences seem to become most manifest. Those frequency measures describe how often a number is used by subjects to complete some task. Several such measures were taken. First, we determined response frequencies of all numbers 1-100 in continued association. A comparison was then made with general language frequency scores, which were available for the numbers 1-20 and the multiples of ten. For these 28 numbers the correlation between Dutch language frequency and psychological frequency, as measured by continued association, was .89. Response frequencies obtained in the two discrete association experiments of Chapter 3 were found to have similar distributions. Small numbers strongly dominated, while tens were also overrepresented. This general picture did not change when, in Chapter 4, people chose numbers to represent some subjective categories, such as pleasant, unpleasant, excitable or calm. In all these instances frequency was found to be a negative function of size. Also, the function seemed to be logarithmic rather than linear. In the continued association experiment almost fifty percent of all named numbers were single digit numbers. In the later experiments this percentage was even higher. It is evident from these findings that size constrains the content of numerical thought.

The very name of the "size effect" seems to invite confusion about its origin and status. "Size" is clearly an important non-mental attribute of numbers. Representing magnitude is precisely what numbers are for. This easily leads to the assumption that size-related information must somehow be responsible for psychological size effects. Size effects must then be special to numbers, since other communicative symbols such as words cannot be ordered on that particular dimension. Our findings strongly suggest, however, that most size effects are not caused by magnitude information. Indeed, such effects seem to result from lack of mental structure, rather than from any specific mental form or content. Thus viewed, all domains of knowledge will have their "size-effects", going by different names. Some people, some cities and some mathematical concepts will always be more familiar than others. Such knowledge seems to represent our cognitive home base, to which we habitually return to make sense of new experience and novel information. In numbers, cognitive familiarity is a negative function of size. That, of course, makes numbers special. But from a psychological point of view the size effect is not as special as its name suggests. Ashcraft recently proposed it should be changed to "size or difficulty effect" (Ahscraft, 1992). This could be further amended. Perhaps its full name should be "size or difficulty resulting from low familiarity and/or low meaningfulness effect".

Frequency and familiarity are good predictors of people's performance in all sorts of tasks. The familiarity of words is usually determined by having subjects rate words on scales, ranging from very familiar to completely unknown. The frequency of words is measured by general language counts. The present study demonstrates that experimental frequency measures have similar predictive qualities. It was found, for example, that associative frequency as measured by continued association was the best single predictor ($r = .78$) of memory scores. This particular frequency variable has several attractions. It can be obtained just by taking the traditional measure of meaningfulness in reverse, and counting how often people use a word, or number, as a response in a continued association task. During continued association, subjects need not choose between associations, but may write down as many as they can think of. Its result is a more complete inventory of knowledge than is obtained by discrete association, which only allows one response per stimulus. It is a more direct measure of knowledge than are language counts and familiarity ratings, which also is an advantage. Moreover, it offers some interesting clues to the structure of personal knowledge.

Frequent numbers seem to function as "reference concepts" - the term was introduced by Rosch - to numerical knowledge and thought. Frequent numbers are primarily single digit numbers, but also some teens and tens. These numbers are most often used to make up new categories, as was shown in Chapter 4. That they are also easier to remember was demonstrated in Chapter 5. Conceptual relationships between numbers are strongly asymmetrical, as demonstrated in Chapter 3. Reference numbers were found to be part of the meaning of many other numbers.

An indication that other fields of knowledge may be similarly structured was obtained by a rough count of associative frequency scores of words. It was found that some words are used with disproportionate frequency, and that these very frequent words stand for very useful things. It would be interesting to explore this issue in other domains of knowledge. Of course, cognitive usefulness should not be confused with ecological usefulness. On the other hand, cognitive development seems to be inherently attuned to human ecology. We first learn by our senses, and come to understand abstractions only much later.

But subjective frequency is not always a better predictor of task performance than is objective size. In the subtraction experiment reported in Chapter 6 we compared the effects of frequency and size for different numbers in one problem. Size was found to be the better predictor for those numbers on which some further action of composition or decomposition had to be performed. These were always the smallest number in a problem. The size of these small numbers had a strong influence on RT. The influence of the larger numbers was less strong, and it was somewhat better described by these numbers' frequency than by their size.

Such differential effects suggest that number processing cannot adequately be captured in one general factor. This suggestion is further supported by the factor analysis performed in Chapter 5. Different tasks tap different attributes of numbers. Frequency, the largest factor, determines which numbers may come up for inspection. This considered, the complex-processing model of Campbell seems to come closer to psychological reality than the abstract-modular model of McCloskey. According to McCloskey long-term knowledge of numbers can be represented by exact quantity specifications. Our results suggest that the use of such explicit information is associated with counting and other non-automatic processing of numbers.