

Differences in the semantic integration of tabled and non-tabled numbers?¹

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Abstract

This study examined the relationship between effects found in studies of cognitive arithmetic and differences in the psychological representation of individual numbers. In two association experiments data were collected about all numbers from 1 to 100. Numbers which are part of the multiplication tables were predicted to be better integrated semantically. Semantic integration was measured in both continuous and discrete association settings using university students as experimental subjects. The variables taken to indicate semantic integration were meaningfulness, commonality of prime responses, percentage of omissions, reaction times and semantic content of the prime response. Tabled numbers were indeed found to obtain better scores than non-tabled numbers on each of these variables. At the same time, a change in the wording of the instruction was found to enhance semantic responses, favouring tabled numbers in particular.

Studies of numerical competence often focus on the basic operations of addition, subtraction, multiplication and division. In this context, the sets of problems formed by the addition and multiplication of all one digit numbers have been researched particularly well. As is generally known, educated people solve these simple problems not by means of explicit rule-driven construction procedures, such as counting, but by means of an associative process, commonly referred to as direct

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retrieval from memory. Studies of the development and functioning of this automatic numerical knowledge have established some effects, which, according to Ashcraft (1992), any theory of cognitive arithmetic must explain. One of them is the well-known size-effect, describing the phenomenon that solution times slow down as the numbers in a problem become larger. Others concern the effects of semantic relationships on the errors subjects will make. To give an example: in an evaluation task subjects with some experience in multiplication will have more difficulty in saying 'no' to $3 + 4 = 12$ than to $3 + 4 = 10$. These subjects also have a harder time rejecting $7 \times 4 = 21$ than $7 \times 4 = 18$. Effects such as these must be ascribed to semantic interference, that is, to subjects' associative sensitivity to well learned combinations of numbers.

In recent years, some influential models have been presented for both the development and the operation of associative numerical knowledge (cf Ashcraft, 1992; Siegler, 1988). A common aspect of these models is that the semantic concepts they focus on are problems, not individual numbers. This "calculation bias" gives them great power, but also a restricted range, leaving certain numerical competencies unaccounted for. These competencies may not be unimportant, however, in that some of them take precedence over calculation at least developmentally. Whether or not some knowledge of numerosity is inborn (Starkey, Spelke, & Gelman, 1992), it can safely be assumed that, in addition to knowing what $5 + 5$ is, we know what 5 is. We can relate it to other numbers, to a spatial arrangement on a die, for example, or to a temporal pattern. Such knowledge must be considered to be an essential part of the concept of five in semantic memory.

Knowledge of number concepts is subject to development. As Miller and Gelman (1983) have shown, numbers like 2, 3, and 4 are represented differently at different ages. While size is important early on, more sophisticated mathematical properties, such as evenness, become a part of the concept only later. It has also been demonstrated that experience with calculation enhances conceptual stability or commonality over subjects (Miller & Stigler, 1991).

Domain-specific experience, however, will not affect all numbers equally. In a recent publication, Dehaene & Mehler (1992) demonstrated that language frequency tends to be inversely related to numerical magnitude. Experience with small numbers is probably both longer and more varied than with large ones. Additionally, it may be argued that the mathematical products of such small numbers are handled more often than their neighbouring primes, and in more varied contexts. Certain differences in conceptual representation must follow. Some such differences have indeed been demonstrated. When Costermans (1990) studied responses to a large sample of numbers, he found that small and round numbers had more 'factorial' semantic connections than large and irregular ones. Elshout & Milikowski (1990) also found, in a group of adult subjects, that associative connections had come to reflect certain well used mathematical properties.

The aim of this study was to demonstrate that certain effects which are persistently found in studies of cognitive arithmetic are also reflected in the representation of the numbers concerned. Focussing on the set of numbers 1 through 100, we were particularly interested in differences between "tabled numbers"

(denoting one digit numbers and their tabled products), and "other numbers" (denoting primes and other non-tabled numbers between 1 and 100). Thus, table membership (of the multiplication tables 1 through 12) was the independent variable throughout, dividing our stimuli into two fairly equal groups of 53 tabled versus 47 non-tabled numbers.

In two association experiments with adult subjects we studied some aspects of these numbers' semantic connections. If our notion is right, tabled numbers should be better integrated semantically; that is they should give rise to more semantic associations than other numbers. Also, the semantic connections of tabled numbers should be stronger; that is reaction times for producing associations should be shorter, while between subjects commonality of associations should be higher.

Experiment 1: Continued Association

Method

The first experiment was conducted to test the hypothesis that tabled numbers have more connections in semantic memory than non-tabled numbers. The critical dependent variable in this experiment was a number's m-score (m standing for meaningfulness, see Noble, 1963), which measures the range of a stimulus' associative connections. M-scores for all numbers 1 through 100 were collected in a continued association setting, in which subjects are required to write down as many associations as possible to a given stimulus in a fixed time.

Subjects. Subjects in this first experiment were eighteen first year psychology students fulfilling a course requirement. Subjects were tested in a group. The duration of the experiment was about an hour.

Materials. The materials consisted of booklets with a separate page for every number. All booklets contained all numbers 1-100, but the order in which the numbers were presented was different for each subject. The stimulus-numbers were printed at the top of the page and also at the beginning of each line to discourage chaining responses.

Procedure. Testing took place in a classroom. After the subjects were seated, the experimenter read out the instruction, which was adopted from Noble (1963), with a few changes. In the current experiment, stimuli were numbers instead of words. Responses also had to be numbers. Subjects were given thirty seconds per stimulus for their responses. As in Noble's instruction, subjects were explicitly warned against chaining responses, and advised to think back to the stimulus number each time before writing down a new response. After the instruction was read out, the booklets were distributed. Following a sign from the experimenter, subjects started on their first number. Time was kept by means of a stop-watch. After fifty numbers a short break was taken.

Results

For all numbers, m-scores were computed, averaged over subjects. The results are shown in Table 1. Tabled numbers gave rise to significantly more associations than non-tabled numbers, $F(1, 98) = 30.68, p < .0001$.

Table 1. Mean m-scores and standard deviations of two classes of numbers: tabled and non-tabled numbers.

	<u>n</u>	Mean m	Std. dev.
Tabled			
tens	10	8.9	.70
one digit-nrs	9	8.41	.32
other	34	8.2	.64
Total	53	8.39	.66
Non tabled			
primes	22	7.63	.57
others	25	7.74	.63
Total	47	7.69	.60

Dividing the main categories of tabled and non-tabled numbers into five sub-categories: tens, one-digit numbers and other tabled numbers on the one side, and primes and other non-tabled numbers on the other (see Table 1), provided some further information. In a comparison of the five sub-category means most differences within the main categories failed to reach significance. Only the mean m-scores of tens and "other" tabled numbers differed significantly. In contrast, when categories on different sides of the main border are compared, all differences were significant on an alpha level of .05.

Discussion

When tabled and non-tabled numbers are compared in terms of their semantic integration as assessed through continuous association, tabled numbers score better. This seems to indicate that tabled numbers have more connections in semantic memory than non-tabled ones. However, a stimulus-number's m-score only tells us how many associations it gives rise to averaged over subjects. It does not tell us anything about the degree of overlap between the responses of different subjects. In theory, all eighteen subjects could have responded to a given number with a completely different set of responses. A number which is responded to in such a seemingly random manner could not really be considered well connected semantically. The reverse could also, theoretically, have happened: all subjects could have responded with exactly the same set of associations to a stimulus number. Such a number would seem to have a completely fixed meaning. The response distributions in our experiment will probably fall somewhere between these two extremes.

In our second experiment, the relative frequency of a number's leading associative response, called its Commonality, was taken as an indication of its stability. The other measures of semantic integration we used were percentage of

missed responses (Omissions), and mean reaction times (RT's). Scores on these three measures were collected in a discrete association experiment. In such a setting, subjects give just one, their first, association to a stimulus - in this case a number. According to our reasoning, tabled numbers should be better integrated semantically. If this is true they should give rise to less random responding and, thus, to more commonality, or associative overlap, between subjects. In general, these better integrated tabled numbers should also be easier to respond to, which should be apparent when RT's and percentage of omissions of tabled and non-tabled numbers are compared.

To complement these measures, we also obtained some information about the content of subjects' responses. A initial inspection of the responses obtained in the continued association experiment suggested that often the preferred response is a small number, which is in some way a component of the stimulus-number. Such small number-responses can be traced to a limited set of relational categories (see also Elshout & Milikowski, 1990; Costermans, 1990). Of these, only the divisor-responses – for example 3 to 9 – were of interest to us here. As the stimulus-category of non-tabled numbers contains many primes, it was easy to predict that it would include fewer divisors among its associates. However, while table membership is thus almost guaranteed to make a difference, we do not know how large this is. Neither do we know if, and to what degree, associative content is related to form of distribution.

Experiment 2: discrete association

The discrete association experiment we have conducted had two parts, each with a different instruction. In the first part of the experiment, subjects were told to respond to a stimulus number, as quickly as possible, with any other number that came to mind. Because subjects clearly found this very difficult, a modified instruction was written and used during the second part of the experiment. In the following, the two instructions will be referred to as 'difficult instruction' and 'easy instruction' respectively. The easy instruction told subjects to respond to a stimulus number as quickly as possible with another number which they felt to be related to it. As the new instruction was meant to influence responses, instructional format was treated as a between-subjects variable in the analyses. The within-subjects variable was, again, table-membership of the stimulus numbers.

Method

Stimuli. As in experiment 1, stimuli were all numbers 1-100.

Subjects. 104 first year psychology students, fulfilling a course requirement, participated as subjects in this experiment, in individual sessions. 54 subjects participated under the "difficult" instruction, and 50 under the "easy" instruction. All subjects were presented with a complete set of stimuli. The duration of the experiment was about 15 minutes.

Apparatus. The experiment was run on a Macintosh Plus ED computer. The stimuli were presented on the monitor screen, in a random order which was newly created during each session. Before the appearance of a stimulus a warning beep

sounded. Responses of subjects were typed on the computer's keyboard out of view from the subject. The reaction times (RT's) of the subject's responses were recorded by means of a voice-key. When a response was registered by the voice-key's microphone the stimulus-number automatically disappeared. The next trial was started when the experimenter pressed a key.

Procedure. After the subject sat down in front of the computer screen, the instruction was read out. Subjects in the first group were instructed to respond to each number appearing on the screen "with any other number, the first that came to mind" as quickly as possible. The second group was instructed to respond to the stimulus, as quickly as possible, "with a number that was related to it in their minds". Apart from this, the instruction was identical for both groups. If no association occurred to a subject, he or she could say so, after which an omission would be recorded. An omission was also automatically recorded when no response was given within five seconds.

After the instruction, subjects were presented with ten practice stimuli, which had been drawn from the set of numbers between 101 and 200. During these practice trials, the voice key was also tested. If necessary, its position or sensitivity was adjusted.

Results

Response distribution. For all hundred number-stimuli, mean scores for both groups of subjects were calculated on the three dependent variables: commonality, or stability of meaning, defined as the percentage of subjects giving the most frequent associative response; omissions, defined as the percentage of subjects failing to respond, and reaction times, which were calculated after eliminating the RT's of the omissions. Mean scores of tabled and non-tabled numbers on these measures in both instructional conditions (difficult and easy) were compared in three two-way analyses of variance (ANOVA's).

On all three measures there was a significant main effect of table membership, a significant main effect of instructional condition, and a significant interaction effect.

Mean scores and standard deviations are presented in Table 2. Results will now be given for the three measures separately.

Table 2. Mean scores and standard deviations of tabled and non-tabled numbers on four measures: meaningfulness, commonality, omissions and RT's. Overall scores are given, together with those of both instructional conditions.

	Tabled (n = 53)		Non-tabled (n = 47)	
<u>Meaningfulness</u>	8.4	(.66)	7.7	(.60)
<u>Commonality</u>				
Overall:	23.7	(6.7)	16.4	(4.7)
Difficult instruction	18.1	(4.8)	14.2	(4.4)
Easy instruction	29.3	(9.9)	18.6	(7.0)
<u>Omissions</u>				
Overall	7.8	(3.2)	10.8	(3.4)
Difficult instruction	11.8	(5.5)	13.4	(3.8)
Easy instruction	3.9	(2.3)	8.1	(4.2)
<u>Reaction Times</u>				
Overall	1386	(74)	1258	(88)
Difficult instruction	1269	(79)	1334	(72)
Easy instruction	1248	(117)	1437	(123)

Tabled numbers had significantly higher commonality-scores than non-tabled numbers ($F(1, 98) = 39.10, p < .0001$). The percentage of omissions was also significantly lower in the tabled condition ($F(1, 98) = 20, p < .0001$). Furthermore, reaction times of responses to tabled numbers were also significantly shorter than of those to non-tabled numbers ($F(1, 98) = 59.9, p < .0001$).

The effect of instructional condition was also significant for all three response distribution measures. The easier instruction had the effect of enhancing commonality ($F(1, 98) = 110, p < .0001$); lowering omissions ($F(1, 98) = 175.4, p < .0001$) and -- unexpectedly -- increasing RT's ($F(1, 98) = 10.5, p = .002$).

On all three measures significant interactions of table-membership and instructional condition were obtained ($F(1, 96) = 19.85, p < .0001$ for commonality, $F(1, 98) = 6.8, p = .01$ for omissions, and $F(1, 96) = 28, p < .0001$ for RT's). An inspection of the data given in Table 2 suggests that all three interactions can be traced to the same cause: tabled numbers benefited significantly more from the change of instruction than non-tabled numbers.

Table 3. Percentage of leading divisor-responses for tabled and non-tabled numbers in both instructional conditions.

	Tabled (n = 53)	Non-tabled (n = 47)
<u>Divisor responses</u>		
Overall	80%	12%
Difficult instruction	77%	2%

Semantic content: The principal associates of each stimulus number are given in the Appendix.

Main effects on semantic content were revealed for both table membership ($F(1, 98) = 117.12, p < .0001$) and instructional condition, ($F(1, 98) = 11.66, p = .0009$), but the interaction just failed to reach significance ($F(1, 98) = 3.67, p = .058$). As Table 3 shows, the preferred type of association of tabled numbers was a divisor in most cases. For non-tabled numbers factorial relationships were scarce or virtually absent among the prime responses.

Discussion

Responses to tabled numbers were given more quickly, with fewer omissions and with more agreement between subjects than responses to non-tabled numbers. Furthermore, this greater ease and commonality of responding was associated with differences in the semantic content of the stimulus-response relationship. Tabled numbers were predominantly responded to with divisors, suggesting factorial relations to be important semantically. For both groups of stimuli, small numbers were the preferred responses. When no divisor is at hand (as in the case of primes), the most frequent response tends to be one of the stimulus' composing digits rather than one of its multiples.

Interestingly, the change we introduced in the instruction seemed to have worked in the same overall direction, raising commonality, lowering omissions and changing semantic content in favour of divisor-responses. Subjects responded more readily and less randomly, thus strengthening existing semantic response patterns. This is clearly demonstrated by the interaction effects: on all measures tabled numbers were affected more strongly by the change of instruction than non-tabled numbers.

At first sight, the variable RT seems to be an exception to the general facilitating influence of the easy instruction. As can be seen by inspection of Table 2, the modified instruction led to slightly longer RT's, an effect we thought rather surprising. Of course, "mean score" is a deceptive measure here. A glance at the data presented in Table 2 confirms that only non-tabled numbers had higher mean RT-scores in the easy condition; tabled numbers are, on the contrary, responded to more quickly. However, the higher mean RT's in the "easy" condition may be an artifact. As stated in the method-section, reaction times were calculated after eliminating the RT's of the omissions, which are, almost by definition, long. As Table 2 shows, the easy instruction led to a substantial decline in omissions - and thus to a decline in eliminated long RT's. In fact, when we compare mean scores calculated on the basis of all RT's, without any rejection of trials, the pattern changes. Mean RT's in the easy and difficult conditions become 1396 msec and 1457 msec respectively, revealing a main effect in the direction we originally expected ($F(1, 98) = 11.00, p = .001$). Mean RT's of tabled and non-tabled numbers were 1383 and 1477 respectively, which is also a significant difference ($F(1, 98) = 12.15, p = .0007$). And again, there was a significant interaction ($F(1, 96) = 6.85, p = .01$), based on cell-means of 1482

msec for the non-tabled and difficult condition, 1472 for non-tabled and easy, 1437 for tabled and difficult, and 1330 for tabled and easy. It is the familiar pattern again: tabled numbers are more strongly affected by the change in the instruction than non-tabled numbers.

How does number association compare with word association? When we compare the scores collected in our experiments with those De Groot (1989) gave for different categories of words, the numbers in our study seem to be less well integrated semantically. However, scores of tabled numbers in the easy condition match those of words quite well. For commonality, De Groot gave mean percentages varying between 23.1 to 36.4; tabled numbers score a percentage of 29.3. Omissions in De Groot's experiment varied between 0.5 and 6.0 percent; tabled numbers had an omission percentage of 3.9 in the easy condition. Reaction times in our experiment were around a hundred milliseconds shorter on the average.

Can the psychological differences between numbers demonstrated here contribute to the understanding of automatic calculation? Of the effects mentioned by Ashcraft (1992), both the error and the relatedness effects appear, to some degree, to be built into the number-concepts as such. Semantic patterns seem to govern the processing of numbers, even when these numbers are not presented in the context of a problem.

Ashcraft's third effect, the problem size or difficulty effect, cannot really be accounted for by the data presented in this paper. However, the predominance of small number responses may be a promising lead for future research.

References

- Ashcraft, M.H. (1992). Cognitive arithmetic: A review of data and theory. Cognition, 44, 75-106.
- Costermans, J. (1990). Les associations entres les nombres de 0 à 1.000.000 en fonction de l'age. Archives de psychologie, 58, 3-27.
- De Groot, A.M.B. (1989). Representational aspects of word imageability and word frequency as assessed through word association. Journal of Experimental Psychology: Learning, Memory and Cognition, 15, 824 - 845.
- Dehaene, S., & Mehler, J. (1992). Cross-linguistic regularities in the frequency of number words. Cognition, 43, 1-29.
- Elshout, J.J., & Milikowski, M. (1990). De produktie van geautomatiseerde rekenkennis [The production of automatic calculating knowledge]. In M. Boekaerts en E. de Corte (Eds.), Onderwijsleerprocessen, ORD, 1990 (pp 199-209). Nijmegen: ITS.
- Miller, K.F., & Gelman, R. (1983). The child's representation of number: A multidimensional scaling analysis. Child Development, 54, 1470 -1479.
- Miller, K.F, and Stigler, J.W (1991). Meanings of Skill: Effects of Abacus Expertise on number representation. Cognition and Instruction, 8, 29-67.

Noble, C.E. (1963). Meaningfulness and familiarity. In C.N. Cofer & B.S. Musgrave (eds.), Verbal behavior and learning: problems and processes. New York: McGraw-Hill.

Siegler, R.S. (1988). Strategy choice procedures and the development of multiplication skills. Journal of Experimental Psychology: General, 117, 258-175.

Starkey, P., Spelke, E. S., & Gelman, G. (1992). Numerical abstraction by human instructional conditions.

Appendix

Principal associations of the numbers 1-100 in four conditions: 1. Continued association overall, 2. Continued association first named, 3. Discrete association with difficult instruction and 4. Discrete association with easy instruction. In cases of a draw for first position between two or three numbers, competitors are listed. When more than three compete, no prime associate is considered to have emerged.

<u>Number</u>	<u>Cont. overall</u>	<u>Cont. 1st named</u>	<u>Discr. difficult</u>	<u>Discr. easy</u>
1	11	11	2	2
2	4	4	1	1
3	9	9	9	9
4	2	2	2	2
5	25	10	6	10
6	3	2	3	3
7	21	14	8	3
8	2	2 / 64	4	4
9	3	3	3	3
10	100	20	1	1
11	1	22	1	1
12	2	24	3	6
13	26	26	-	1 / 3 / 7
14	7	7	7	7
15	5	30	5	5
16	8	1 / 4	8	4
17	1	1	7	3 / 18
18	9	9 / 36 / 1	9	9
19	20	20 / 1	20	20

Appendix continues

<u>Number</u>	<u>Cont. overall</u>	<u>Cont. 1st named</u>	<u>Discr. difficult</u>	<u>Discr. easy</u>
20	4	2 / 40	2 / 10	10
21	3	1 / 2 / 42	7	7
22	11	2	11	11
23	2	46	7 / 25	7
24	2	48	12	12
25	5	5 / 50	5	5
26	2	2	4	4
27	9	2 / 9	9	9
28	2	2	2	7
29	30	30	30	1 / 30
30	3	60	3	10
31	1	3 / 62	4 / 30	3
32	16	64	8	8
33	66	3 / 66	3	3 / 11
34	3	3	7 / 12 / 35	6
35	70	70	5	5
36	6	6	6	6
37	10	3	3	7
38	2	3	40	2 / 8
39	3	40	40	3
40	20	20	10	10
41	1	82	5	5 / 9
42	21	4 / 24	6 / 7	8
43	3	4	7	7
44	11	11	4	11
45	90	90	9	5

Appendix continues

<u>Number</u>	<u>Cont. overall</u>	<u>Cont. 1st named</u>	<u>Discr. difficult</u>	<u>Discr. easy</u>
46	23	4	8 / 10	4
47	11	4	7	7
48	12	4	12	8 / 12
49	7	7	50	7
50	5	100	5 / 100	5
51	102	5	50	9
52	2	5 / 104	7	8
53	106	5	8	7
54	9	108	9 / 55	6
55	5	5	5	5
56	4	5	-	-
57	12	5	60	7
58	8	5	-	2
59	60	60	60	60
60	6	6	6 / 10	10
61	6	1 / 6 / 62	-	9
62	8	6 / 8 / 31	8 / 60	8
63	9	6	7	7
64	2	8	8	8
65	5	6	11	5
66	33	6 / 36	6	6
67	7	6	13	7
68	34	6	8 / 70	8
69	9	6	70	13
70	7	7	10	10
71	1	8	8	72

Appendix continues

<u>Number</u>	<u>Cont. overall</u>	<u>Cont. 1st named</u>	<u>Discr. difficult</u>	<u>Discr. easy</u>
72	9	7 / 36	8	8
73	7	7	7	7
74	4	7	75	6 / 7
75	25	7 / 150	5 / 25	25
76	6	7	7	4 / 7 / 13
77	7	7	7	7 / 11
78	8 / 87	7	80	2
79	97	80	80	8 / 80
80	8	8	8	10
81	9	18	9	9
82	41	8	8	2
83	11	8	8	7
84	4	8	8	6
85	5	8	15	15
86	43	8	8	4
87	7	8	90	3
88	8	8	8	8
89	9	8	90	9
90	9	9	10	10
91	9	9	9	9
92	9	11	8	8
93	3	9 / 12	7	7
94	4	9	6 / 95	6
95	5	9	100	5
96	3	9	100	4
97	16	9	3	3

Appendix continues

<u>Number</u>	<u>Cont. overall</u>	<u>Cont. 1st named</u>	<u>Discr. difficult</u>	<u>Discr. easy</u>
98	8	9	100	2
99	33	100	100	9
100	10	1	10	10