CHAPTER I Two faces of knowledge

This dissertation offers a description of a limited body of knowledge. It shows what ordinary educated adults know - and do not know - about the numbers 1-100. The experimental measures are concerned with the content, as well as the form, of people's number-related thoughts. Both are clearly relevant to knowledge, in which the "what" and the "how" must cooperate to produce meaningful behaviour. For instance: it is no less important to know which numbers people name in response to some question, than it is to know how long it takes them to do so. The human subjects in the experiments are first year students of psychology. These people are taken to represent "the average adult". The more alike they are the better it is from our point of view, since the real subjects of this study are not people, but numbers. These individual entities, the numbers 1 to 100, are the ones to be tested and examined in various experiments. Questions will be addressed such as: how do these numbers relate to each other according to people's knowledge, how frequently are they used, how well are they remembered and what do people feel about them. The dissertation could thus be viewed as a case study, aiming at a multi-faceted description of a well-defined population of familiar mental objects.

One of the difficulties in understanding knowledge is that it is subject to dual control, and has been so during the entire history of its formation. In what we call knowledge two systems are involved, which work by completely different rules. At the one end, there is flesh and blood: a head filled with billions of neurons, which have their own methods and pathways of communication. Neurons respond to other neurons, but have no a priori sensitivity to letters, words, numbers and other symbols of human communication. At the other end there are textbooks, recipes, methods and grammars, describing how things are from a non-mental point of view. Here, knowledge is organized in terms of its own rules, meanings and relationships. That two plus two makes four is a mathematical truth, which our biology has come to "know" somehow.

Formal knowledge must be implemented in a system which, as such, is blind to the intrinsic rules and contents of that knowledge. This is what Skinner called: bringing behaviour under control of a stimulus configuration. For complex forms of abstract knowledge the establishment of such control demands years of work. Children go to school for ten or twenty years. When education fails, culture loses its grip on its members. The same thing happens when the biological hardware breaks down: words and objects lose control over the minds of demented people.

When Skinner spoke about the establishment of stimulus control, he operated from the principle that people can be trained to adapt themselves to almost any type of externally made up configuration. The method he developed was a form of step by step programming of new forms of behaviour, called operant conditioning. Skinner was in many respects a successful operant conditioner. During the war he even secretly trained pigeons to serve as bombardiers, and though these pupils never served in any actual mission, they were clever performers by the available reports.

Some of the problems with the approach became apparent only later. In an amusing paper called "The misbehaviour of organisms" two of Skinners former collaborators reported about unexpected breakdowns on an animal training farm (Breland & Breland, 1961). Well trained animals would sometimes suddenly, and stubbornly, "regress" to natural habits which supposedly had been overcome. Raccoons would rub the experimental coins together instead of putting them in the experimental piggy banks. Hens would ferociously pick at plastic pellets supplied for quite different purposes. It seemed like a revolt of nature. The authors, who had been dedicated behaviourists, concluded that behaviour is less adaptable than they had formerly thought.

Findings of Garcia and his colleagues , who worked with rats, also underlined that not all things can be learned with equal facility. They had rats drink "bright-noisy" saccharine-sweetened water, while exposing them to imperceptible X-rays which made them sick after some hours. Garcia found that rats developed an aversion to the taste, but not the "bright-noise" associated with the water. Evidently the hypothesis of a sick rat is: "it must be something I ate", wrote the experimenters (Garcia & Koelling, 1974, p. 504). This and other findings (see Seligman, 1974) led to the notion of "preparedness" and "contra preparedness" which is now taken up again in the context of evolutionary approaches to psychology (e. g., Geary, 1995; Scarr, 1994; Sloboda, 1992). Animals, including humans, are evidently biased in their treatment of information. The circumstance that things are coupled in time and place does not mean that brains will always associate them. Skinners theory of the formation of knowledge is clearly much too simple. The brain seems to have its own intrinsic preferences for processing information. Some experiences leave a strong mark while others do not. Some stimuli are of immediate interest, while others may only become so after much thought and practice.

This does not mean, however, that Skinner's notion of "stimulus control" as established by experience and education is not a sensible one. If a person habitually thinks and responds in terms of established knowledge, he or she may well be viewed as, in some sense, operating under control of that knowledge. People who speak a language are no longer free not to understand what is said to them. For people who can read it is also very difficult not to perceive the meaning of a string of letters, which "obviously" forms a word. Minds are thus colonized by the culture that raises them. One point of view is to say it makes one unfree. Another is to say it makes one free to deal with things that could not be dealt with otherwise. These viewpoints are not necessarily contradictory. Babies as young as a few month have been irreversibly captured by the sounds of their own language: they can no longer perceive some foreign language distinctions which they were sensitive to at birth. Cats raised in a basket painted in vertical stripes will be functionally blind to other patterns for the rest of their lives. In the case of the cats this is a real handicap. In the case of the babies it is the price paid for the acquisition of useful abilities. That numbers have come to "control us", in the sense that we cannot help recognizing 10 when we perceive it, is primarily a good thing considering that the alternative is inaptitude rather than freedom (compare Keil, 1989).

Cognitive psychology has been called "mentalist" by Skinner and others because it placed the structures governing behaviour inside people's heads instead of in the environment. But Herbert Simon, one of the founders of cognitive psychology, firmly believed - and presumably still believes - that the structure of thinking primarily reflects the properties of the objects that are thought about. In one of his books, "The sciences of the artificial" (1969, 1981), Simon compared the mental steps people take when solving a puzzle to the traces left by an ant which has to cross a rough patch of beach to reach its home. The pattern the ant leaves is obviously a reflection of its behaviour. But its complexity "is really a complexity in the surface of the beach, not a complexity in the ant." The same can be said about human behaviour, mental or otherwise. Its apparent complexity "is largely a reflection of the complexity of the environment in which [a person] finds himself" (Simon, 1981, pp. 64-65). That human problem solving protocols - the verbal traces of thought - can indeed be mapped on the structure of the problems that are tackled has since been amply demonstrated. New materials are "internalized" step by step, with many detours and fallbacks, because people cannot take in and retain large amounts of new information. The very limited capacity of short term memory may be compared to the ant's short legs.

But Simon's theory deals with performance in the absence of knowledge, rather than with the structure of knowledge that has been acquired. After some practice, many problems cease to be problematic. While ants' legs do not grow with experience, short term memory capacity does, as measured by its grasp on the material at hand. The "chunks" by which short term memory can be measured - the term was invented by Miller (1956) and adapted by Simon - are abstract entities whose empirical content is, at all times, a function of the task in combination with long term knowledge.

In fact two different forms of "stimulus control" seem to be involved. The kind of external control Simon refers to is an indirect one: it is concerned with the processes of mental adaptation. The other more direct type of control can be viewed as its result. When it has been established, some meanings need no longer be spelled out but are perceived at once.

The process is well illustrated by the development of calculation. Children first solve problems by means of counting. The steps they take can be easily traced, and mapped on the structure of the task. But after some time a different type of control takes over. Simple problems, which have been frequently met and successfully solved, are now understood and responded to in a seemingly automatical way. The answers to such problems just present themselves, independent of the subject's conscious intentions. This type of development occurs in many fields of knowledge and ability (Ericsson & Smith, 1992; Van Lehn, 1989). Expert performers automatically think, reason and act in terms of the domain's prescriptions and demands (e. g. Boshuizen & Schmidt, 1992; De Groot, 1946, 1965; Chi, 1978; Chi, Feltovitch & Glaser, 1981; Chi, Glaser & Farr, 1988; Gobbo & Chi, 1986; Schraagen, 1993, 1994).

By some appearances the structure of well developed knowledge has thus become isomorphic to the rules and content of the domain in question. Nature's initial preferences and obstacles seem to have been successfully overcome, suggesting that knowledge can now be described in terms of its non-mental properties only. This is the essence of many of the "conceptual network models" by which knowledge has been represented (cf. Anderson, 1990; Smith, 1989).

There are several reasons to suppose, however, that such an approach takes too much for granted. It suggests that psychological meaning - or content - has now some permanent existence within the mind, independent of its manifestations. This is a problematic assumption. When meaning is always there - though perhaps not always easily accessible - the thing to be explained is the absence of its manifestations rather than their presence. This seems to be starting from the wrong end. Also, the notion of permanently built-in meanings makes it difficult to distinguish between items. For example, how many numbers should be assumed to have permanent presence in the mind, and by which empirical criteria is the question to be decided? And if it has been decided, and unknown and known numbers have been somehow definitely separated, it still leaves the main problem unsolved. Not all of the "known" numbers can be expected to be equally well known by any empirical measure. How to represent such differences in degree of meaning and/or accessibility?

There is an alternative approach to the representation of knowledge which avoids some of these difficulties. According to Barsalou (1987, 1989, 1993), "concepts" should not be viewed as permanent mental entities, but as products of mental activity in a certain context. Concepts, as viewed by Barsalou, have no definite mental form. They are produced in working memory to serve some goal. Long term knowledge and task demands decide on its form. This approach has several attractions. It avoids many of the conceptual puzzles inherent to the notion of permanent meanings. By defining concepts as products of the interaction of external prescriptions and internal facilities it offers a better opening for an understanding of the dynamics of dual control.

The literature on mental number-processing and calculation contains many indications of a highly complex knowledge structure. Even simple and frequently used numbers are not known equally well." There is also evidence that the degree to which numbers are "well known" is strongly - and negatively - related to their objective size. The phenomenon has been most extensively studied in the context of simple calculation (e. g. Ashcraft, 1982, 1992; Groen & Parkman, 1972; Hamann & Ashcraft, 1985; Parkman & Groen, 1971). Even adult subjects, who solve many of the basic combinations (additions and multiplications of single-digit numbers) seemingly automatically, do not solve them with equal speed or reliability. Though a problem such as 8 + 7 is no longer counted out, and no discrete steps can be distinguished in the construction of its solution, this problem takes significantly longer to solve than a smaller sized problem, such as 3 + 7. People also make more mistakes in such larger problems. A partial explanation of this phenomenon has been offered by Ashcraft and his collaborators, who found that small sized problems are overrepresented in primary school textbooks (Ashcraft, 1992; Hamann & Ashcraft, 1986). Tasks that have been tackled more frequently are generally performed more quickly.

Size effects, however, are not only found in calculation studies. They are found in almost any task involving numbers. One simple example is subitizing, the immediate perception of numerosity. Though adults subitize better than children, their ability to "see" how many dots or matches a pattern contains is nonetheless restricted to very small quantities. Patterns of more than four objects are difficult to discriminate by subitizing (Chi & Klahr, 1975; Krueger, 1982). Such examples point to an inherent advantage of small sized quantities.

That small sized quantities are not only easier to perceive but also easier to reason with was demonstrated in a study by Winer (1974). This author discovered that the ability of very young children to perform some tasks of logical reasoning (Piagetian conservation) depended on the quantities involved. Ideas about "more" and "less" were more correctly applied to smaller than to larger numbers. This result can be compared with the inability of the average adult to appreciate the meaning of very large numbers. Many people will sometimes make a stronger distinction between one hundred and two hundred than between one billion and two billion (Paulos, 1988). Such large numbers apparently cannot be given adequate meaning to. Interestingly, certain relative size-related inadequacies of number-knowledge seem to be a problem even at high levels of expertise. A famous mental calculator once observed that "for him, every number up to a thousand was but one idea, and every number between a thousand and a million was, to his regret, two ideas" (Hunter, 1977, p. 43).

The relative poorness of knowledge associated with larger numbers may underlie differences found on more implicit measures. An example of such implicit differences can be found in a study by Banks and Hill, who investigated the structure of people's mental number-lines. One interesting finding was that subjects implicitly gave more space to smaller than to larger numbers. The relationship between objective size and awarded space could be described by a logarithmic function (Banks & Hill, 1974; see also Banks &

Coleman, 1981).

Armstrong, Gleitman and Gleitman (1983) also compared numbers on an implicit psychological measure. Their study concerned typicality-differences within categories. In that context, some sets of numbers were also tested. Subjects were presented with several pairs of numbers. In a pair, the numbers were both odd, or both even. Subjects' task, when shown such pairs, was to indicate which of the two numbers was the "more typically odd", or the "more typically even" exemplar. The aim of the study was to obtain a clarification of the typicality gradient itself, which had been discovered by Eleanor Rosch (Rosch, 1973, 1978) as an interesting property of knowledge. Prior to the Armstrong et al. study, differences in typicality were considered by some psychologist to reflect objective differences in concepts' well-definedness. To test this explanation Armstrong and her colleagues presented their subjects with sets of well-defined concepts. The results showed that objective well-definedness did not affect typicality gradients. Odd and even numbers, which are well-defined categories, were nonetheless reliably rated by people as being more or less "typically" odd or even. Particularly interesting in this context is the table containing the concepts and their scores. It shows that a perfect negative correlation was obtained between typicality and size. The smaller the number, the higher its typicality score (Armstrong et al., 1983, p. 276).

It is easier to see that things are so than to understand why they are so. On the one hand, there is a biological system which may have its initial preferences - or disinterests - with respect to different numbers and the quantities they stand for. In a recent paper Geary (1995) suggested that some numerical abilities will develop in any society, because they have relatively strong foundations in human biology (see also Gallistel & Gelman, 1992; Starkey, Spelke & Gelman, 1990; Wertheimer, 1938). Other abilities may only develop if people are persistently "force-fed" by formal instruction. At the same time, this "force-feeding", as practised by modern societies, also discriminates between numbers. The textbooks with their bias against larger-sized arithmetic problems are one example. An even more telling example, because it covers such a large terrain, are the frequency counts collected by Dehaene and Mehler (1992). Comparing frequency distributions of numbers and number words in seven languages, these authors found that small numbers occur much more often in each of these.

They also observed, however, that the correlations between numerical size and language frequency is anything but perfect. Numbers such as 10, 12, 20, 60 and 100 are overrepresented comparatively. This suggests that special properties of numbers may make their own contribution to the formation of knowledge. Of course, they can only do so by the mediation of experience. But education offers much experience with various formal properties of numbers. That properties such as evenness gradually become part of long-term knowledge of different numbers was demonstrated by a study of Miller and Gelman (1983). These authors found that, while young children base their "similarity" judgements primarily on size, older children prefer to use different distinguishing features, such as evenness. Other examples may be found in studies of mental calculation. When people confuse numbers, these are often numbers belonging to a special subcategory. Errors in multiplication, for instance, are predominantly "wrong multiples". Thus, 36 (4 x 9) may be confused with 24 (4 x 6), or with 32 (4 x 8), but not with its immediate neighbours, such as 37, or 25, or 33 (Campbell & Clark, 1988, 1992; Campbell & Graham, 1985). And in a preliminary study of 10 year old children solving story problems, we found that certain number pairs (for instance 60 : 20, or 72 : 12), were quickly perceived in terms of their multiplicative relationships, while others (54 : 18) were not. Such quick perceptions facilitated reasoning about these problems (Milikowski, 1988). These findings indicate that other attributes than size may influence the degree to which numbers are known and understood.

At the start of this introduction I stated that the subjects of this study are one hundred mental "things", being the numbers 1-100. The choice to study this particular set of numbers deserves some explanation. In the psychology of language it is usual to compare psychological responses to different groups of words which have a certain attribute in common. Thus, frequent words may be compared with non-frequent words, or long words with short words. In such contexts little attention will be paid to the individual characteristics of a stimulus. This is different, however, when the objects of investigation are semantic categories. In such cases words are selected for their meaning. Numbers can be viewed as a very large semantic category, with infinitely many members. It is obvious that we cannot study them all. To select the first hundred, the numbers 1-100, seemed a reasonable choice. These are, presumably, the numbers most people know best. Within this set of hundred numbers many comparisons can be made. Studying a fixed selection of mental objects was also attractive to us because it offers an opportunity to combine two approaches. General comparisons can be made, while at the same time descriptive information is obtained, showing how specific numbers score on specific measures.
In this book several experimental psychological measures will be taken of the numbers 1-100. Throughout the study, experimental scores of numbers will be compared with some of their objective attributes. The main objective attributes used in these comparisons are size (standing for objective magnitudes of numbers), tabledness (distinguishing between numbers which do, and numbers which do not, occur in the multiplication tables 1-12), certain subcategories of tabled and non-tabled numbers, and evenness (distinguishing between even and odd numbers). The experimental measures will be introduced in the relevant chapters.

1 A reference to Skinner's own book: The Behaviour of Organisms (1938).